TRANSPORT PROCESSES IN MULTICOMPONENT GAS–LIQUID MIXTURES

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UDC 532.533

Transport processes in multicomponent mixtures of dense gases and liquids are described. The method for describing transport processes in dense media first proposed in [1] is used.

1. Initial System of Equations and Formulation of the Problem. To describe transport processes in multicomponent dense mixtures we will use the following model:

The particles of the medium are considered structureless with a mass m_i and diameter σ_i ($i = 1, 2, \ldots, m$) is the mixture component number where m is the number of components in the mixture;

All mixture components are characterized by the same temperature T and mass velocity $\mathbf{u}$;

We consider paired particle interaction. The potential of paired intermolecular interaction of particles of types i and j, $\phi_{ij}(\mathbf{r}_i, \mathbf{r}_j)$ is definable as a sum of solid sphere potentials and the long-range attractive portion of the intermolecular potential $\Phi_{0ij}(\mathbf{r}_i, \mathbf{r}_j)$ [2]:

$$\varphi_{ij}(\mathbf{r}_i, \mathbf{r}_j) = \begin{cases} \varphi_{\text{T.c}ij}(\mathbf{r}_i, \mathbf{r}_j), & |\mathbf{r}_i - \mathbf{r}_j| < \sigma_{ij}, \\ \Phi_{0ij}(\mathbf{r}_i, \mathbf{r}_j), & |\mathbf{r}_i - \mathbf{r}_j| \geqslant \sigma_{ij}. \end{cases} \tag{1.1}$$

Here $\mathbf{r}_i$, $\mathbf{r}_j$ are spatial coordinates of type i and j particles; $\sigma_{ij} = (\sigma_i + \sigma_j)/2$ is the distance between type i and j particles upon contact.

The basic equation for the single-particle distribution function $f_i(\mathbf{r}_i, v_i, t)$ of mixture component i has the form

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \frac{\partial f_i}{\partial \mathbf{r}_i} + \frac{\mathbf{F}_i}{m_i} \cdot \frac{\partial f_i}{\partial \mathbf{r}_i} - m_i^{-1} \sum_{j=1}^{m} \int\int \frac{\partial \Phi_{0ij}(\mathbf{r}_i, \mathbf{r}_j)}{\partial \mathbf{r}_i} \cdot \frac{\partial f_{2ij}}{\partial \mathbf{v}_i} d\mathbf{r}_j \, d\mathbf{v}_j = J_{\text{ci}}, \tag{1.2}$$

where $\mathbf{v}_i$ is the velocity of type i particles; t is time; $\mathbf{F}_i(\mathbf{r}_i, t)$ is the external force acting on a type i particle; $f_{2ij}(\mathbf{r}_i, \mathbf{v}_i, \mathbf{r}_j, \mathbf{v}_j, t)$ is the two-particle distribution function:

$$f_{2ij} = f_i f_j G_{ij}.$$

Here $G_{ij}(\mathbf{r}_i, \mathbf{r}_j, t)$ is the paired particle pair-correlation function; J_{ci} is the collision integral in the rigid sphere model:

$$J_{\text{ci}} = \sum_{j=1}^{m} \sigma_{ij}^2 \int\int_{(\mathbf{v}_{j,i} \cdot \mathbf{e}) \geqslant 0} (\mathbf{v}_{j,i} \cdot \mathbf{e}) \{ f'_{2ij}(\mathbf{r}_i, \mathbf{v}'_i, \mathbf{r}_i + \sigma_{ij}\mathbf{e}, \mathbf{v}'_j, t) - \\ - f_{2ij}(\mathbf{r}_i, \mathbf{v}_i, \mathbf{r}_i - \sigma_{ij}\mathbf{e}, \mathbf{v}_j, t) \} d\mathbf{e} \, d\mathbf{v}_j;$$

the prime denotes the value of functions after collision; $\mathbf{e}$ is a unit vector; $\mathbf{v}_{j,i} = \mathbf{v}_j - \mathbf{v}_i$.

In representing molecular interactions in the form of Eq. (1.1) the collision integral appearing in Eq. (1.2) can be written in the form of the sum of contributions from particle interactions with the rigid sphere potential $\phi_{rsij}(\mathbf{r}_i, \mathbf{r}_j)$ and contributions from particle interactions with the long-range attractive portion of the intermolecular potential defined by the last term on the left side of Eq. (1.2). A similar representation of the collision integral was considered in [2] and justified in [3].

Taking for the paired correlation function G_{ij} its equilibrium value G_{0ij} and considering the central character of the intermolecular potential $\Phi_{0ij}(\mathbf{r}_i, \mathbf{r}_j)$, we write the left side of Eq. (1.2) as

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Kiev. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 6, pp. 122-128, November-December, 1991. Original article submitted November 24, 1988; revision submitted July 19, 1990.

0021-8944/91/3206-0934$12.50

$$Df_i \equiv \frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \frac{\partial f_i}{\partial \mathbf{r}_i} + m_i^{-1}\left(\mathbf{F}_i - \frac{\partial \mathcal{U}_{\mathrm{efi}}}{\partial \mathbf{r}_i}\right) \cdot \frac{\partial f_i}{\partial \mathbf{v}_i} \tag{1.3}$$

$$(\mathcal{U}_{\mathrm{efi}}(\mathbf{r}_i, t) = n_i(\mathbf{r}_i, t) \int \Phi_{0ij}(\mathbf{r}_i, \mathbf{r}_j)\, G_{0ij}(\mathbf{r}_i, \mathbf{r}_j, t)\, d\mathbf{r}_j),$$

and the right side in the form

$$\begin{gathered} J_{\mathrm{ci}} = \sigma_{ij}^2 \iint\limits_{(\mathbf{v}_{j,i}\cdot \mathbf{e}) \geqslant 0} (\mathbf{v}_{j,i} \cdot \mathbf{e}) \{G_{0ij}(\mathbf{r}_i, \mathbf{r}_i + \sigma_{ij}\mathbf{e}, t) \times \\ \times f_i'(\mathbf{r}_i, \mathbf{v}_i', t)\, f_j'(\mathbf{r}_i + \sigma_{ij}\mathbf{e}, \mathbf{v}_j', t) - G_{0ij}(\mathbf{r}_i, \mathbf{r}_i - \sigma_{ij}\mathbf{e}, t) \times \\ \times f_i(\mathbf{r}_i, \mathbf{v}_i, t)\, f_j(\mathbf{r}_i - \sigma_{ij}\mathbf{e}, \mathbf{v}_j, t)\}\, d\mathbf{v}_j\, d\mathbf{e}. \end{gathered} \tag{1.4}$$

Transforming to dimensionless variables in Eq. (1.2) and equating Eqs. (1.3) and (1.4) we obtain

$$Df_i = \varepsilon^{-1} J_{\mathrm{ci}}. \tag{1.5}$$

Here $\varepsilon = (nr_0^2 L)^{-1} = \ell/L = \mathrm{Kn}$; ε is a parameter defining the degree of system inhomogeneity, having the order of magnitude of the gradients of the hydrodynamic variables; n is the numeric particle density; r_0 is the effective intermolecular interaction radius; L is the characteristic linear macroscopic flow dimension; $\ell = (nr_0^2)^{-1}$; Kn is the Knudsen number. For practically all flows of dense gas and liquid this parameter is small ($\varepsilon \ll 1$) and thus in finding the solution of system (1.5) in the hydrodynamic regime one can construct that solution as an expansion in the small parameter ε.

2. Hydrodynamic Asymptote of the Solutions of System (1.5). System (1.5) can be solved by the Chapman–Enskog method [4], where for the small parameter ε we use gradients of the hydrodynamic variables. We seek a solution in the hydrodynamic regime in the form

$$f_i = f_i(\mathbf{r}_i, \mathbf{v}_i, \Gamma_i(\mathbf{r}, t)) = \sum_{n=0}^{\infty} \varepsilon^n f_i^{(n)}(\mathbf{r}_i, \mathbf{v}_i, \Gamma_i(\mathbf{r}, t)) \tag{2.1}$$

($\Gamma_i(\mathbf{r}, t)$ are the mean values of the hydrodynamic variables).

Substitution of Eq. (2.1) in the expression for J_{ci}, Eq. (1.4), and expansion of the integrand terms therein in a Taylor series in the vicinity of the point with coordinate $\mathbf{r}_i$ leads to an expression for the right side of Eq. (1.5) in the form of a series in powers of ε:

$$J_{\mathrm{ci}} = \sum_{n=0}^{\infty} \varepsilon^n J_{\mathrm{ci}}^{(n)}. \tag{2.2}$$

We substitute Eq. (2.1) into Eq. (1.3) and expand the evolution operator in a series in the small parameter ε. This reduces Eq. (1.3) to the form

$$Df_i = \sum_{n=0}^{\infty} \varepsilon^n (Df_i)^{(n)}. \tag{2.3}$$

With consideration of Eqs. (2.2) and (2.3), Eq. (1.2) takes on the form

$$\sum_{n=0}^{\infty} \varepsilon^n (Df_i)^{(n)} = \varepsilon^{-1} \sum_{n=0}^{\infty} \varepsilon^n J_{\mathrm{ci}}^{(n)}. \tag{2.4}$$

Now in Eq. (2.4) we equate terms with identical powers of the parameter ε, obtaining equations for finding the quantities $f_i^{(n)}$, which appear in Eq. (2.1), defining the single-particle distribution function.

We will solve Eq. (2.4) in the zeroth approximation in hydrodynamic quantity gradients: $0 = J_{\mathrm{efi}}^{(0)}$ This solution, which is a local equilibrium solution of Eq. (1.2), can be written as

$$\ln f_{ip}(\mathbf{r}_i, \mathbf{v}_i, t) = m_i \alpha_i^{(1)} + \mathbf{v}_i \cdot m_i \alpha_i^{(2)} + \left(\frac{m_i v_i^2}{2} + \mathcal{U}_{\mathrm{efi}}(\mathbf{r}_i, t)\right) \alpha_i^{(3)},$$

where $\mathcal{U}_{\text{ef }i}(\mathbf{r}_i, t)$ is the interaction energy of particle i with all mixture particles; $\alpha_i^{(1)}, \alpha_i^{(2)}, \alpha_i^{(3)}$ are constants subject to definition.

As a result we have

$$f_{ip}(\mathbf{r}_i, \mathbf{v}_i, t) = \bar{f}_i(\mathbf{r}_i, \mathbf{v}_i, t)\exp\{-\mathcal{U}_{\text{ef}i}(\mathbf{r}_i, t)/kT(\mathbf{r}, t)\}.$$

Here k is Boltzmann's constant; $\bar{f}_i(\mathbf{r}_i, \mathbf{v}_i, t)$ is the local equilibrium Maxwell distribution function; $T(\mathbf{r}, t)$ and $\mathbf{u}(\mathbf{r}, t)$ are the local values of temperature and hydrodynamic velocity. Solubility of the inhomogeneous linear integral equations of system (2.4) requires orthogonality of the invariants of the collision integrals J_{ci} to the inhomogeneous portion of integral equations of Eq. (2.4).

In the zeroth approximation in thermodynamic quantity gradients we obtain Euler type hydrodynamics equations, in which the new expressions for the pressure $p(\mathbf{r}, t)$ and the specific heat of the mixture at constant volume $c_V(\mathbf{r}, t)$ have the form

$$p = \sum_{i=1}^{m} n_i kTJ_i\left[1 + \frac{2\pi}{3}\sum_{j=1}^{m} n_j\sigma_{ij}J_j \times G_{0ij}(\sigma_{ij})\right] - \frac{2\pi}{3}\sum_{i,j=1}^{m} n_in_jJ_iJ_j\int\Phi'_{0ij}(R)G_{0ij}(R)R^3\,dR, \tag{2.5}$$

$$c_V = \frac{3}{2}k + (2nT)^{-1}\sum_{i,j=1}^{m} n_in_jJ_iJ_j\int\Phi_{0ij}(R)G_{0ij}(R)\,d\mathbf{R}, \quad \mathbf{R} = \mathbf{r}_i - \mathbf{r}_j,$$

$$J_i(\mathbf{r}, t) = \exp\left\{-(kT)^{-1}\sum_{j=1}^{m} n_j(\mathbf{r}_i, t)\int\Phi_{0ij}(\mathbf{r}_i, \mathbf{r}_j)G_{0ij}(\mathbf{r}_i, \mathbf{r}_j, t)\,d\mathbf{r}_j\right\};$$

$G_{0ij}(\sigma_{ij})$ is the value of the function at the point of contact of particles of types i and j; function arguments are omitted where possible.

We write the solution of Eq. (1.2) in the approximation linear in thermodynamic quantity gradients for a single-particle distribution function as

$$f_i = f_{ip}[1 + \Phi_i], \tag{2.6}$$

$$\Phi_i = \mathbf{A}_i\cdot\nabla\ln T - \widehat{B}_i : \nabla\mathbf{u} - n\sum_{k=1}^{m}\mathbf{G}_i^k\cdot\mathbf{d}_k.$$

Here

$$\widehat{B}_i = B_i^{(1)}\left(\mathbf{W}_i\mathbf{W}_i - \frac{1}{3}W_i^2U\right) + B_i^{(2)}U;$$

$$\mathbf{A}_i = \sum_{r=0}^{\infty} a_{ir}S_{3/2}^{(r)}(W_i^2)\,\mathbf{W}_i; \quad B_i^{(1)} = \sum_{r=0}^{\infty} b_{ir}^{(1)}S_{5/2}^{(r)}(W_i^2); \; B_i^{(2)} = \sum_{r=0}^{\infty} b_{ir}^{(2)}S_{1/2}^{(r)}(W_i^2);$$

$$\mathbf{G}_i^k = \sum_{r=0}^{\infty} g_{ir}^{(k)}S_{3/2}^{(r)}(W_i^2)\,\mathbf{W}_i; \quad \mathbf{W}_i = (m_i/2kT)^{1/2}(\mathbf{v}_i - \mathbf{u});$$

U is a unit tensor; $S_m^{(n)}(W_i^2)$ are Sonin polynomials [4]; $\mathbf{d}_k(\mathbf{r}, t)$ is the diffusion thermodynamic force of mixture component k, defined in the usual manner [4], with new values for the mixture pressure and specific heat, Eq. (2.5).

The system of linear equations for finding the coefficients a_{ir}, $b_{ir}^{(1)}$, $b_{ir}^{(2)}$, $g_{ir}^{(k)}$ has the form

$$\sum_{j=1}^{m}\sum_{q=0}^{N-1} L_{ij}^{pq}a_{jq}(N) = -\frac{15}{4}n_iJ_i(2kT/m_i)^{1/2}\left[1 + \frac{8\pi}{5}\sum_{j=1}^{m}\sigma_{ij}^3n_jJ_jG_{0ij}(\sigma_{ij}) \times m_{ij}m_{ji}\right]\delta_{p1},$$

$$\sum_{j=1}^{m}\sum_{q=0}^{N-1} H_{ij}^{(1)pq}b_{jq}^{(1)}(N) = 5n_iJ_i(2kT/m_i)^{1/2}\left[1 + \frac{8\pi}{15}\sum_{j=1}^{m}\sigma_{ij}^3n_jJ_jG_{0ij}(\sigma_{ij})m_{ji}\right]\delta_{p0},$$

$$\sum_{j=1}^{m}\sum_{q=0}^{N-1} H_{ij}^{(2)pq}b_{jq}^{(2)}(N) = n_iJ_i(2kT/m_i)^{1/2} \times \left[1 + \frac{4\pi}{3}\sum_{j=1}^{m}\sigma_{ij}^3n_jJ_jG_{0ij}(\sigma_{ij})m_{ji} - \frac{3}{2}p(nTc_V)^{-1}\right]\delta_{p1},$$

$$\sum_{j=1}^{m}\sum_{q=0}^{N-1} L_{ij}^{pq} g_{ir}^{(k)} = \frac{3}{2}(2kT/m_i)^{1/2}(\delta_{ik} - \rho_i/\rho)\,\delta_{p0}, \tag{2.7}$$

$$p = 0, 1, 2, \ldots, N-1, \quad m_{ij} = m_i/(m_i + m_j),$$

where $\sum\limits_{q=0}^{N-1}$ denotes the N-th order approximation in Sonin polynomials. The quantities appearing in Eq (2.7) are defined by the relationships

$$L_{ij}^{pq} = \delta_{ij}\sum_{l=1}^{m} n_i J_i n_l J_l \left[S_{3/2}^{(p)}(W_i^2)\,\mathbf{W}_i;\ S_{3/2}^{(q)}(W_i^2)\,\mathbf{W}_i\right]'_{il} +$$

$$+ n_i J_i n_j J_j \left[S_{3/2}^{(p)}(W_i^2)\,\mathbf{W}_i;\ S_{3/2}^{(q)}(W_j^2)\,\mathbf{W}_j\right]''_{ij}, \quad L_{ij}^{pq} = L_{ji}^{qp}, \quad \sum_{i=1}^{m} L_{ij}^{0q} m_i^{1/2} = 0,$$

$$H_{ij}^{(1)pq} = \delta_{ij}\sum_{l=1}^{m} n_i J_i n_l J_l \left[S_{5/2}^{(p)}(W_i^2)\left(\mathbf{W}_i\mathbf{W}_i - \frac{1}{3}W_i^2 U\right);\ \ S_{5/2}^{(q)}(W_i^2)\left(\mathbf{W}_i\mathbf{W}_i -\right.\right.$$

$$\left.\left.- \frac{1}{3}W_i^2 U\right)\right]'_{il} + n_i J_i n_j J_j \left[S_{5/2}^{(p)}(W_i^2)\left(\mathbf{W}_i\mathbf{W}_i - \frac{1}{3}W_i^2 U\right),\right.$$

$$\left. S_{5/2}^{(q)}(W_j^2)\left(\mathbf{W}_j\mathbf{W}_j - \frac{1}{3}W_j^2 U\right)\right]''_{ij},$$

$$H_{ij}^{(2)pq} = \delta_{ij}\sum_{l=1}^{m} n_i J_i n_l J_l \left[S_{1/2}^{(p)}(W_i^2),\ S_{1/2}^{(q)}(W_i^2)\right]'_{il} + n_i J_i n_j J_j S_{1/2}^{(p)}(W_i^2),$$

$$S_{1/2}^{(q)}(W_j^2)\Big]''_{ij},$$

$$\sum_{i=1}^{m} n_i a_{i0} m_i^{1/2} = 0, \quad b_{i0}^{(2)} = 0,$$

$$\sum_{i=1}^{m} n_i b_{i1}^{(2)} = 0, \qquad \sum_{i=1}^{m} n_i g_{ir}^{(k)} m_i^{1/2} = 0$$

(the symbol [;] denotes integral brackets [4]).

3. Transport Coefficients of a Dense Medium. The single-particle distribution function (2.6) permits determination of the transport coefficients of a dense medium (shear viscosity η_1, volume viscosity η_2, thermal conductivity λ, mutual diffusion D_{ij}, barodiffusion D_i^p, thermodiffusion ratio K_i^T, thermodiffusion D_i^T):

$$\eta_1 = \frac{kT}{2}\sum_{i=1}^{m} n_i b_{i0}^{(1)} J_i \left[1 + \frac{8\pi}{5}\sum_{j=1}^{m} n_j J_j \sigma_{ij} m_{ji} G_{0ij}(\sigma_{ij})\right] +$$

$$+ \frac{8}{15}\sum_{i,j=1}^{m} (2\pi m_{ji} m_i kT)^{1/2} n_i n_j \sigma_{ij}^4 J_i J_j G_{0ij}(\sigma_{ij}),$$

$$\eta_2 = \frac{4}{9}\sum_{i,j=1}^{m} (2\pi m_{ji} m_i kT)^{1/2} n_i n_j \sigma_{ij}^4 J_i J_j G_{0ij}(\sigma_{ij}) +$$

$$+ \frac{2\pi}{3}\sum_{i,j=1}^{m} \sigma_{ij}^3 m_{ji} n_i n_j J_i J_j G_{0ij}(\sigma_{ij})\, b_{i1}^{(2)},$$

$$\lambda = \frac{5}{4}(2k^3T^3)^{1/2}\sum_{i=1}^{m} n_i m_i^{-1} J_i \left[1 + \frac{8\pi}{15}\sum_{j=1}^{m} \sigma_{ij} n_j J_j m_{ij}\, m_{ji} G_{0ij}(\sigma_{ij})\right]\left(a_{i1} -\right.$$

$$\left. - \sum_{l=1}^{m} g_{i1}^{(l)}\tilde{d}_l\right) + \left(\frac{4}{3}\,2\pi k^3T^3\right)^{1/2}\sum_{i,j=1}^{m} \sigma_{ij}^4 n_i n_j G_{0ij}(\sigma_{ij})\, J_i J_j\, (m_{ij} m_{ji}/(m_i + m_j))^{1/2},$$

$$D_{ij} = \frac{\rho_i J_i}{2m_j}(2kT/m_i)^{1/2}\sum_{k=1}^{m} g_{i0}^{(k)}(E_{kj} - P_j E_{kl}/P_l),$$

$$D_j^p = (\rho_j J_j/2\rho)(2kT/m_j)^{1/2} \sum_{k=1}^{m} g_{j0}^{(k)} E_{kl}/P_l kT,$$

$$D_i^{\mathrm{T}} = (\rho_i J_i/2\rho)(2kT/m_i)^{1/2} \Bigg[\sum_{k=1}^{m} g_{i0}^{(k)} \Bigg\{ \sum_{j=1}^{m} n_k J_k \left(\delta_{kj} + \frac{4\pi}{3} n_j J_j \sigma_{kj}^3 m_{kj} G_{0kj}(\sigma_{kj}) \right) +$$

$$+ n_k J_k n_j J_j T \left(\int \nabla\Phi_{0kj}(\mathbf{r}, \mathbf{r}_j) \nabla_{\mathrm{T}} G_{0kj}(\mathbf{r}, \mathbf{r}_j)\, d\mathbf{r}_j + \frac{2\pi}{3} \sigma_{kj}^3 \nabla_{\mathrm{T}} G_{0kj}(\sigma_{kj}) \right) -$$

$$- E_{kl} P_l^{-1} \sum_{p=1}^{m} \Bigg[n_p J_p \left(\delta_{pj} + \frac{4\pi}{3} n_j J_j \sigma_{pj}^3 m_{pj} G_{0pj}(\sigma_{pj}) \right) +$$

$$+ n_p J_p n_j J_j \left(\int \nabla\Phi_{0pj}(\mathbf{r}, \mathbf{r}_j) \nabla_{\mathrm{T}} G_{0pj}(\mathbf{r}, \mathbf{r}_j)\, d\mathbf{r}_j + (2\pi/3)\, \sigma_{pj} \nabla_{\mathrm{T}} G_{0pj}(\sigma_{pj}) \right) \Bigg] \Bigg\} - a_{i0} \Bigg],$$

$$K_l^{\mathrm{T}} = -(P_l n_l J_l)^{-1} \Bigg\{ \sum_{j=1}^{m} (1 - \delta_{jl}) P_j n_j J_j K_j^{\mathrm{T}} -$$

$$- \sum_{i,j=1}^{m} \Bigg[n_i J_i \left(\delta_{ij} + \frac{4\pi}{3} n_j J_j \sigma_{ij}^3 m_{ij} G_{0ij}(\sigma_{ij}) \right) +$$

$$+ n_i J_i n_j J_j T \int \nabla\Phi_{0ij}(\mathbf{r}, \mathbf{r}_j) \nabla_{\mathrm{T}} G_{0ij}(\mathbf{r}, \mathbf{r}_j)\, d\mathbf{r}_j + \frac{2\pi}{3} \sigma_{ij}^3 \nabla_{\mathrm{T}} G_{0ij}(\sigma_{ij}) \Bigg] \Bigg\}.$$

Here $\tilde{d}_l$ is defined by the expression

$$\sum_{l=1}^{m} \left(g_{i1}^{(l)} - g_{i1}^{(h)} \right) (1 - \delta_{lh}) \tilde{d}_l = a_{i0} \quad (i \neq h), \quad E_{hj} = \delta_{kj} +$$

$$+ \frac{4\pi}{3} \sigma_{jk}^3 J_k G_{0kj}(\sigma_{kj}) + \frac{2\pi}{3} \sum_{i=1}^{m} n_k J_k \sigma_{ki}^3 n_i J_i \nabla_{n_j} G_{0ki}(\sigma_{ki}),$$

$$P_l = \sum_{i=1}^{m} \Bigg[\delta_{il} kT + (4\pi/3)\, \sigma_{li}^3 n_i J_i kT G_{0il}(\sigma_{il}) +$$

$$+ (2\pi/3) \sum_{j=1}^{m} \sigma_{ij}^3 n_i J_i n_j J_j kT \nabla_{n_l} G_{0ij}(\sigma_{ij}) \Bigg] + \sum_{i=1}^{m} n_i J_i \left(I_{il} + I_{li} + \sum_{j=1}^{m} n_j J_j \nabla_{n_l} I_{ij} \right),$$

$$I_{ij} = \int \nabla\Phi_{0ij}(\mathbf{r}, \mathbf{r}_j)\, G_{0ij}(\mathbf{r}, \mathbf{r}_j)\, d\mathbf{r}_j.$$

With consideration of Eq. (3.1) the expressions for the momentum flux tensor $\Pi(\mathbf{r}, t)$, the heat flux vector $\mathbf{Q}(\mathbf{r}, t)$, the mass flux vector $\mathbf{J}_i(\mathbf{r}, t)$ of mixture component i have the form

$$\Pi = pU - 2\eta_1 \left\{ \hat{\varepsilon} - \frac{U}{3} \nabla \cdot \mathbf{u} \right\} - \eta_2 U \nabla \cdot \mathbf{u},$$

$$\mathbf{Q} = -\lambda \nabla T + \sum_{i=1}^{m} h_i \mathbf{J}_i + p \sum_{i=1}^{m} K_{\mathrm{T}i} \mathbf{J}_i / \rho_i,$$

$$\mathbf{J}_i = -\sum_{j=1}^{m} (1 - \delta_{ji}) D_{ij} m_j \nabla (n_j/n) - D_i^{\mathrm{T}} \rho_i \nabla \ln T - \rho_i D_i^p \nabla \ln p,$$

where $\hat{\varepsilon}(\mathbf{r}, t)$ is the deformation rate tensor; $\rho_i(\mathbf{r}, t)$ is the mass density of component i; h_i is the specific partial enthalpy.

We note that the initial generalization of the Enskog equation to the case of a mixture [4] had the shortcoming that the transport coefficients did not satisfy the Onsager relationships [5]. This fault was corrected in [6].

In the present study satisfaction of the Onsager reciprocity relationships is ensured by correct choice of the expressions for the thermodynamic quantity fluxes and thermodynamic forces which minimize the entropy production for steady state processes. To do this it is necessary to use the equilibrium paired correlation distribution function which is a nonlocal functional of the thermodynamic variable fields. Thus in the approximation linear in thermodynamic quantity gradients the kinetic equation will contain an additional term in the collision integral. Consideration of such terms, as was shown in [6], permits solutions which agree with the thermodynamics of nonequilibrium processes and ensures satisfaction of the Onsager reciprocity relationships.

The expressions for the transport coefficients obtained in the present study agree with transport coefficients found with the approach of [6] in the limit in which the attractive portion of the intermolecular potential $\Phi_{0i}(\mathbf{r}_i, \mathbf{r}_j)$ may be neglected.

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